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# HANDLING UNCERTAINTY IN INPUT TO EXPECTED VALUE MODELS

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Due to the large number of entities and processes that must be represented, combat models at the theater level in the Army today are expected value models. An expected value model is deterministic--it uses the expected value of random variables as inputs and generally uses some sort of expected value within the internal processes. The use of expected value models creates problems in the proper interpretation of their output and ways for representing the uncertainty associated with the model and input and processes.

This paper suggests a method for handling uncertainty in the input data sets (which usually contain elements that are specific realizations of random processes) in situations where the outcomes of interest can be expressed in binary variables (e.g., "success" or "failure"). A theater nuclear exchange is used as an example, having many different possible outcomes determined by random processes. A method is provided for describing the space of all possible outcomes of the exchange and partitioning the space into sets of outcomes which, if used as input into a theater-level conventional simulation, are expected to lead to significantly different results. A method for sampling the most probable outcome from each set is also explained.

This approach permits the construction of an experimental plan that requires a small number of model runs, each run expected to provide a significantly different result. From these runs an estimate of the variability in the theater combat resulting from uncertainty in the input data (in this case, the impact of a nuclear exchange) can be made.

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# HANDLING UNCERTAINTY IN INPUT TO EXPECTED VALUE MODELS

September 1989

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ABSTRACT. Due to the large number of entities and processes that must be represented, combat models at the theater level in the Army today are *expected value* models. An expected value model is deterministic -- it uses the expected value of random variables as inputs and generally uses some sort of expected value within the internal processes. The use of expected value models creates problems in the proper interpretation of their output and ways for representing the uncertainty associated with the model input and processes.

This paper suggests a method for handling uncertainty in the input data sets (which usually contain elements that are specific realizations of random processes) in situations where the outcomes of interest can be expressed in binary variables (e.g., "success" or "failure"). A theater nuclear exchange is used as an example, having many different possible outcomes determined by random processes. A method is provided for describing the space of all possible outcomes of the exchange and partitioning the space into sets of outcomes which, if used as input into a theater-level conventional simulation, are expected to lead to significantly different results. A method for sampling the most probable outcome from each set is also explained.

This approach permits the construction of an experimental plan that requires a small number of model runs, each run expected to provide a significantly different result. From these runs an estimate of the variability in the theater combat resulting from uncertainty in the input data (in this case, the impact of a nuclear exchange) can be made.

THE RESEARCH SPONSOR was the Director, US Army Concepts Analysis Agency (CAA).

THE OBJECTIVE OF THE RESEARCH was to develop a method for summarizing a stochastic process as input into a deterministic expected value model.

THE MAIN ASSUMPTIONS used in this research were:

(1) The stochastic process (outcome of a nuclear exchange) can be summarized as sets of binary variables, each variable indicating the defeat or failure to defeat a unit.

(2) It is possible to identify significantly different outcomes of a nuclear exchange in terms of sets of defeated and nondefeated units.

(3) The probabilities of defeating units can be evaluated independently.

THE BASIC APPROACH used in this research was to use the probability that a targetable subunit (such as a company or battery) can be defeated (which can be obtained from the Theater Analytic Nuclear (TACNUC) Model under development at CAA) to determine the probability that units (such as divisions) represented in the theater-level expected value model can be defeated. The space of all possible outcomes of the exchange is described in terms of binary variables representing the defeat or failure to defeat a unit. The outcome space can be partitioned into sets of outcomes which, if used as input into a theater-level conventional simulation, will lead to significantly different results. From these partitions an experimental plan is constructed that identifies a small number of outcome realizations of the exchange to use as input to the expected value model, running the model once for each input.

THE PRINCIPAL FINDING of the research is that is is possible to summarize a stochastic process such as the outcome of a theater nuclear exchange as input to a deterministic conventional model of battle at the theater level.

THE RESEARCH WAS PERFORMED BY MAJ Mark A. Youngren, Requirements Directorate, CAA.

COMMENTS AND QUESTIONS may be sent to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-RQN, 8120 Woodmont Avenue, Bethesda, MD 20814-2797.

# CONTENTS

PARAGRAPH | Page



## FIGURES

# *HANDLING UNCERTAINTY IN INPUT TO EXPECTED VALUE MODELS*

## *Introduction*

Modeling large systems and processes such as combat at the theater level is difficult. The number of possible units and interactions has driven most modelers to use an *expected value* approach. An expected value model uses the expected value of random variables as inputs and generally uses some sort of expected value within the internal processes. The models are *deterministic*; that is, they will yield only one set of outputs for any given set of inputs. The use of expected value models creates problems in the proper interpretation of their output and ways for representing the uncertainty associated with the model input and processes. In a recent discussion paper, Stockton [1989] provided the following example:

"A Red unit will go northwest or northeast based on whether his strength at a given point is above or below some threshold value. Let's say that the real-world probability of being above the threshold is 0.6 and, if above, he will go northwest to face a very strong Blue force armed with Supertank. If he goes northeast (probability 0.4), he faces a relatively weaker force, armed with bows and arrows. With several replications of a stochastic model, expected losses will consider both possibilities and will develop expenditures of tank ammo <u>and</u> arrows; with an expected value model, he will <u>always</u> go toward the stronger force, and no expenditures of arrows will be observed."

Stockton correctly points out that the results of an expected value model, even when provided expected value inputs, are <u>not</u> the expected value of the output. He suggests that the output of such a model may be a "most likely value," using his example. However, we can offer another example which illustrates that expected value models also fail to provide a "most likely" result.

Suppose in the example provided above that the Red force has a visual sensor that can see all of the Blue forces traveling together (with probability 1) if the skies are clear, and cannot see any of the Blue force if the skies are cloudy. To simplify, suppose that the skies are either clear or cloudy, and the probability that the skies are clear is 0.6. How many Blue units are detected by the Red force? The expected value is $0.6 \cdot$ (100 percent of the Blue units) $+ 0.4 \cdot$ (0 percent of the Blue units) $=$ 60 percent of the Blue units. Expected value models will normally apply expected values,

either as inputs to the model (60 percent would be an expected value for the probability of target acquisition) or internal to the processes. Note, however, that acquiring 60 percent of the Blue force is the least likely outcome, as it occurs with probability 0! Even if we chose the most likely result of 100 percent detection (which is not the way that expected value models generally handle continuous variables as opposed to choices), we run into problems.

Now let us combine the two examples. It is reasonable to suppose that if the Red force can see the Blue force, or even a large percentage of the force, it will notice that one force is armed with Supertank and the other with bows and arrows. Thus, given detection, it will engage the weaker (bows and arrows) force. If we have the model take the most likely values in the two examples, it will (1) detect 100 percent of the Blue force and (2) go northwest to engage the Blue force. Each result is by itself most likely, yet the result is the most unlikely. Even if one modeled the Red force detection at 60 percent, the combination of a 60 percent detection (still sufficient to distinguish between Supertank and bows and arrows) and moving northwest is unlikely.

Admittedly, these examples are simplistic. Yet it is true that expected value models not only fail to yield the expected value of the output, they also fail to yield the most likely output. What, then, is the probability associated with the output of an expected value model? The answer to that question, unfortunately, is "nobody knows." This is why expected value models can yield counterintuitive, contradictory, and/or nonsensical results when initially tested. The usual approach when this occurs is to adjust input data, processes, thresholds, etc. until the model yields "reasonable" results. Hopefully this yields a model that will provide suitably realistic results with a different input data set, but there are no guarantees. We unquestionably have no way of determining the likelihood of any given output from a complex expected value model.

*Sources of Uncertainty*

There are two areas of uncertainty properly associated with an expected value model that must be handled: uncertainty in the model input, and uncertainty in the model processes.

Unfortunately, a "blessed" input data set is often regarded as certain - if we have approval for a set of numbers to be used in the study, then those numbers are the set to use to support our analysis. Excursions from the base data set for purposes of analysis will vary only a small number of data items by design; the others remain fixed. Some input data values are truly fixed; the air distance from Bremen to Munich is an example. Other values may be fixed by scenario; for example,

the daylight hours vary by latitude and time of year; a scenario will fix a time and place that will in turn determine the appropriate value for daylight. Unfortunately, these scenario-driven items are often fixed arbitrarily, even when they may have an impact upon the analysis. For example, if a force is particularly vulnerable to detection by a sensor that requires daylight, you can get different results in a summer versus winter scenario (which will in turn be different than that obtained using an arbitrary number like 8 hours or 12 hours). This difference may even be apparent in studies that seemingly are not associated with detection -- ammo rates could be significantly different, for example. This is a simple, obvious example; many others, not so easily identified, exist. We must regard the input data set as a single realization of many stochastic variables. It is not always clear which realization to select for use -- averages do not always exist and may not be appropriate. Furthermore, correlations exist between sets of these data inputs; for example, selecting the most likely or expected values of cloud cover and rain independently may yield the combination of sunny with 1 inch of rain! Note that this problem exists with stochastic (Monte Carlo) models -- they also require a fixed data set that is not varied from run to run.

Uncertainty also exists in the model processes. Stochastic models generally handle this uncertainty through random number draws, although they are also subject to problems associated with correlations (separate random number draws generally require independence) and fixed values such as thresholds. The examples provided above illustrate some of the problems associated with handling process and input uncertainty within an expected value model.

### *Addressing Uncertainty in Expected Value Models*

At this point, it would be nice to be able to make a statement like "the solution to this problem is easy; one simply needs to... ." Unfortunately, there are no simple, universal solutions to the problems associated with addressing uncertainty in expected value models. It is clear, however, that any methods that might alleviate the problem must deal with the uncertainty associated with the data input as well as the uncertainty associated with the model processes. Furthermore, the uncertainty in the input data justifies the following assertion: *executing an expected value model only once for a given data set does not provide a meaningful result.* If an expected value model is to be used to support analysis, the user *must* be prepared to execute multiple runs, varying in some meaningful fashion the input data and/or the model processes, in order to establish some measure of the uncertainty associated with the output of such a model.

Ideally, such an approach will minimize the number of runs required (because running a large expected value model can be very costly), yet provide a significantly different result from each run, thus increasing the variance across all outputs. We want to be able to describe the probability that the conditions represented in the input for each run (or conditions similar to those represented) will occur.

We have developed an approach to handling input uncertainty in theater-level expected value models in situations when the outcomes of interest can be expressed in terms of binary variables; i.e., one can describe all events as "yes" or "no," "on" or "off," etc. The particular application that will be developed deals with a theater-level tactical nuclear exchange.

Several models of conventional warfare exist at the theater level. The model used at CAA is called the Force Evaluation Model (FORCEM). Like most theater-level models and scenarios, FORCEM is a low resolution expected value model, representing combat forces at the division and higher level and time in 12-hour steps. The Nuclear Effects Model Embedded Stochastically in Simulation (NEMESIS) research at CAA (Youngren [1989]) documents an analytic model for describing the possible outcomes of a theater-level tactical nuclear exchange. The methodology described in this paper arose from the need to summarize the stochastic outcomes of the theater-level exchange as input to FORCEM.

*The Scenario*

In a theater-level battle where nuclear weapons may be employed, the commander of the forces on a side may have an overall objective (such as stabilizing the forward line of own troops (FLOT) in the defense or achieving a breakthrough in the offense) that will necessitate the use of nuclear weapons. In order to meet this objective, the commander will specify the *defeat criteria* against each unit -- that is, the necessary degree of damage to be achieved against each unit to meet his objective. The defeat criteria will differ from unit to unit depending upon the unit mission, the posture, the equipment, etc. The criteria applied to larger units (such as divisions) will frequently focus fires on critical subordinate units. For example, the defeat criteria for a unit might be achieving a latent lethal dose (about 450 rad) against at least 50 percent of the personnel in the unit. The defeat criteria for a particular division might be to defeat at least 50 percent of the infantry units *or* at least 40 percent of the armor units in the division.

Although the effects of a tactical nuclear laydown at the theater perspective are normally described in terms of defeating divisions, tactical nuclear weapons within the theater are targeted against forces at the company and battery level. The term *subunit* (also target or target subunit) used in this paper denotes a combat organization (such as a company) that would be targeted by a nuclear weapon. The size of the subunit will depend both upon the capabilities of the weapon system used to engage the subunit and the targeting doctrine of the firer. For example, companies may be targeted close to the FLOT using small, artillery-fired weapons, while battalions may be targeted deep using missiles or air-delivered weapons. For purposes of exposition, we will refer to the low-resolution combat organizations represented in theater models such as FORCEM (usually divisions, although other forces may be represented as well) as *units*.

There are very many targetable subunits in a typical theater scenario, on the order of $10^4$. As a result, there are $2^{10^4}$ possible outcomes that can occur in terms of the defeat or failure to defeat each subunit. Even if we look only at the defeat or failure to defeat the low resolution aggregate units represented in our theater model (usually several hundred), we still have on the order of $2^{10^2}$ possible outcomes. Even with sophisticated techniques and considerable confounding, classical experimental design approaches require at least one run per variable. The large amount of time and effort required to execute even a simple run of a typical theater-level expected value model prohibit more than a few model runs for any study. Classical experimental designs therefore obviously cannot be applied. Our objective is to construct a plan that minimizes the number of different input data sets (thus minimizing the number of theater-level model runs) yet fully reflects the range of possible outcomes of the theater nuclear exchange.

### *A Method for Addressing Input Uncertainty in Expected Value Models*

Describing the outcome of the theater-level nuclear exchange on each unit in terms of defeat criteria allows us to define a binary variable $B_i$, where $B_i = 1$ if the unit is defeated; 0 otherwise. Given the assumption that the outcome is independent between units, the outcome of any exchange is simply a set of 0's and 1's with the probability that any $B_i = 1$ equal to $p_{defeat}(i)$, the probability that unit $i$ is defeated, $i = 1, \ldots, m$. Methods for easily calculating the probability of defeat for each targetable subunit are given in Youngren [1989]. Given $m$ units, there are $2^m$ possible outcomes. Clearly, if we define defeat criteria in terms of total numbers of potential nuclear targets (on the order of $10^4$), there are too many outcomes to enumerate.

At the theater level, however, defeat criteria can usually be expressed in terms of divisions and a limited number of other high value targets -- on the order of at most several hundred across a theater. Each division, in turn, will have its defeat criteria established in terms of units subordinate to that division. For example, suppose that a division $j$ has 10 battalions of infantry (engaged as battalions), 24 armored companies (engaged as companies), and 20 batteries of artillery. The defeat criteria for this division may be 50 percent of the infantry, 40 percent of the armor, or 60 percent of both, with a separate criteria for artillery (divisional and nondivisional). In terms of maneuver subunits, 5 infantry battalions or 10 armor companies must be defeated in order to defeat the division. There are $\frac{(10+24)!}{p!\,(10-p)!\,q!\,(24-q)!}$ ways of choosing p infantry battalions and q armored battalions for defeat, and *all* combinations where $p \geq 5$, $q \geq 10$, or $( p + q ) \geq 60$ percent of the subunit (which can be worked out for specific values of p and q ) lead to the defeat of this division. If we assume that each subunit $i$, $i = 1, \ldots, 34$ has a unique probability of defeat $p_{defeat}(i)$, we probably do not wish to enumerate all sets of subunits where the division is defeated and compute the joint probability (which will be the product of $p_{defeat}(i)$ for the subunits $i$ defeated and $(1-p_{defeat}(i))$ for the subunits that are not). Fortunately, this situation is readily amenable to Monte Carlo solutions. We simply need to draw 34 binary pseudorandom numbers $B_i$ such that each number $B_i = 1$ with probability $p_{defeat}(i)$, and let a binary variable, say $D_n$, equal 1 if the set of numbers $B_i$ drawn correspond to division $j$ being defeated, 0 otherwise. If we perform N replications of this experiment, we can estimate P[ division defeated ] $= \frac{1}{N}\sum_{n=1}^{N} D_n$. If we do this for each division $j$, then we have a probability $p_{defeat}(div\ j) \equiv$ P[ division $j$ defeated ] for $j = 1, \ldots, ndiv$, where $ndiv$ = the number of divisions.

At the division level, we can define a binary variable $O_j$ to define the outcome of the nuclear exchange with respect to division $j$, $j = 1, \ldots, ndiv$. $O_j = 1$ with probability $p_{defeat}(div\ j)$ if division $j$ is defeated; 0 otherwise.

Across the theater, the theater commander will desire at least a certain percentage of units be defeated in order for the employment of nuclear weapons to be considered effective. We can define a binary function of the random variables $\underline{O}$, $\phi(\ \underline{O}\ )$, such that $\phi(\ \underline{O}\ ) = 1$ if the commander's objective is met; 0 otherwise. Clearly $\phi(\ \underline{O}\ )$ is nondecreasing in $\underline{O}$. The function $\phi$ may be regarded as identical to a structure function of a coherent system in reliability theory (Barlow & Proschan [1981]); thus we can use results from coherent structure theory in our analysis of the nuclear exchange issue.

For example, if any $k$ out of $m$ divisions must be defeated in order for the commander's objective to be met,

$$\phi(\underline{O}) = (O_1 O_2 \cdots O_k) \coprod (O_1 O_2 \cdots O_{k-1} O_{k+1}) \coprod \cdots \coprod (O_{m-k+1} \cdots O_m),$$

for all possible subsets of size $k$ from the $m$ units, $1 \le k < m$, where

$$(x_j) \coprod (x_i) \equiv 1 - (1 - x_i)(1 - x_j).$$

Furthermore, we can bound $P[\phi(\underline{O}) = 1]$ by (Barlow & Proschan [1981] p. 31):

$$\max_{1 \le r \le npath} \prod_{i \in P_r} P[O_i = 1] \le P[\phi(\underline{O}) = 1] \le \min_{1 \le s \le ncut} \coprod_{i \in K_s} P[O_i = 1],$$

where $P_r$ denotes one of the $npath = \binom{m}{k}$ possible min path sets (in this case, a min path set is any set of $k$ units), $K_s$ denotes one of the $ncut = \binom{m}{m-k+1}$ possible min cut sets (in this case, a min cut set is any set of $m$-$k$+1 units), and $\coprod_i X_i = 1 - \prod_i (1 - X_i)$. If we let $p_o(i) = P[O_i = 1]$, and number the units such that $p_o(1) \le p_o(2) \le \cdots \le p_o(m)$, then

$$\max_{1 \le r \le npath} \prod_{i \in P_r} P[O_i = 1] = \prod_{i=m-k+1}^{m} p_o(i) \; ; \; \min_{1 \le s \le ncut} \coprod_{i \in K_s} P[O_i = 1] = \coprod_{i=1}^{m-k+1} p_o(i).$$

This example of a $k$ out on $m$ defeat criteria shows how we can estimate (through bounds) the probability that the commander's objective may be met. Alternatively, $P[\phi(\underline{O}) = 1]$ can be estimated using the same Monte Carlo technique used to find $P[O_j = 1]$ for each division $j$.

*Partitioning the Space of All Possible Outcomes*

At the theater level with a total of $nt$ division-sized and high value targets, if we examine the nuclear exchange outcome $O_j$ for each division (or equivalent high-value target), there are $2^{nt}$ possible outcomes. It may be the case that it makes a difference in the battle that follows the nuclear exchange *which* units are defeated or targets destroyed in the exchange. Or, more simply, it may be *how many* units are defeated and targets destroyed across the theater which makes a difference.

It is possible to define sets of outcomes of the nuclear exchange that, given our best judgment, we expect to have a significantly different effect on any subsequent theater-level battle (if all outcomes have approximately the same effect, then there is one set consisting of all outcomes). We choose these sets by selecting *partitions* dividing the sample space (space of all possible outcomes) into *strata* such that the following properties are met:

(1) All events within a given stratum will yield approximately the same overall theater-level outcome. As a result of this assumption, we regard all events within any given stratum as *exchangeable.*

(2) Any set of $n$ events from $n$ different strata are expected to yield $n$ different theater-level outcomes. Thus, any pair of events from two different strata are *not* exchangeable.

In practice, all events within a stratum will not be truly exchangeable, and the two events to either "side" of any partition will likely lead to similar theater-level outcomes. Nevertheless, it is possible to conceive of outcome sets with different results, and we assume for all of the development below that these two properties are obeyed.

For example, suppose that there are 20 opposing divisions in a sector of combat. Our best judgment, given the tactical and operational situation, is that the defeat of at least 7 divisions out of the 20 will be required to avoid loss of territory (stabilize the FLOT--which may be the commander's objective). However, if 14 or more divisions are defeated, an opportunity occurs not merely to stabilize the FLOT but also to conduct a successful counterattack. In this case, if $O_i = 1$ if division $i$ is defeated, $i = 1, \ldots, 20$, there are $2^{20}$ possible outcomes. We can partition the sample space of possible outcomes into the $\sum_{k=0}^{6} \binom{20}{k}$ outcomes where 6 or fewer divisions are defeated, the $\sum_{k=7}^{13} \binom{20}{k}$ outcomes where 7 or more but less than 14 divisions are defeated, and the $\sum_{k=14}^{20} \binom{20}{k}$ outcomes where 14 or more divisions are defeated.

The example given above involved two partitions (three strata); the number of partitions required depends on the number of significantly different theater-level outcomes that need to be represented. Selecting the partitions will require experienced judgment and possibly some experimentation with the theater model. If one is unsure about how many partitions to select, the number of strata should equal the maximum number of theater model runs you can afford.

*Stratified Sampling from the Sample Space*

Once the sample space (space of all possible outcomes) has been identified, it is possible to perform a stratified sampling from the sample space, each sample from the outcome of the nuclear exchange model forming an input vector to the theater-level conventional model. From each stratum created by our partitions, a single realization can be sampled. A random sampling approach can be

used; however, since the actual likelihood of all of the events within a stratum may vary widely, we recommend using a fixed sampling scheme, in particular sampling the mode from each partition. Given the assumption of exchangeability between events within a stratum, any choice will have a roughly equivalent effect on the theater-level outcome, so any choice is valid. Using the mode allows us to compensate for the fact that the events within the stratum are only approximately exchangeable. A modal (most likely) outcome will also form a plausible input suitable for subsequent analysis. The theater-level conventional model, such as FORCEM, will be run $ns$ times for each of the $ns$ strata created from $ns-1$ partitions, using the outcome selected from each stratum as an input. If the second assumption that we made in selecting the partitions is met, the $ns$ battles simulated in FORCEM using outcomes from the $ns$ different strata should yield noticeably different results. The response surface estimated using these $ns$ FORCEM runs should provide a better representation of the variability possible in theater-level combat where nuclear weapons are employed than a random selection of $ns$ outcomes from the $2^{nt}$ outcomes possible, where $nt$ is the number of targetable subunits in the theater.

The question naturally arises, "what if I am wrong in selecting the partitions?" Partitioning is a judgemental process; more of an art than a science. The situation in which this technique is to be used is one where many runs of the deterministic model are not possible; therefore, it is not possible to sample the results of many outputs given many different input data sets describing different nuclear exchange outcomes. As a result, we simply do our best to try and force realizations from areas of the space of all possible outcomes where we *think* that the theater-level outcome will be different. The impact of being wrong is not much different than being right. We still have another point in the theater-level outcome space that you are sampling. The fact that the nuclear exchange outcome did *not* lead to the theater-level outcome expected should be of great interest to the analysis. Either the theater model has deficiencies in correctly representing the impact of the exchange, or the theater situation is (surprisingly) robust to the exchange. If the theater outcome that you tried to create (by selecting the nuclear exchange outcome stratum) is still of interest, another run could be attempted (if time and resources permit), sampling from a more extreme point within the stratum.

*Selecting the Most Likely Outcome (Mode) From Each Stratum*

Selecting the mode from each stratum is simple and not computationally intensive. The partitions defining the stratum will establish the outcome vectors $\underline{O}$ that fall within each stratum. Recall that $p_o(j) = P[\ O_j = 1\ ]$, and let $q_o(j) = 1 - p_o(j)$. Order the $p_o(j)$ and $q_o(j)$'s together

from the largest to the smallest value. To select the mode within each partition, go from the first value ( $p_o(j)$ or $q_o(j)$ ) and select the outcome $O_j = 1$ for each $p_o(j)$ and the outcome $O_j = 0$ for each $q_o(j)$. Continue until each target $j$ has an outcome assigned, making sure to assign only one outcome to each target. It will be necessary to "skip" over the higher probability ( $p_o(j)$ or $q_o(j)$ ) for some targets $j$ in order to have a total set of outcomes fall within the partition.

This procedure can most easily be understood through an example. Suppose we have five divisional units with the following probabilities of defeat ( $P[\ O_j = 1\ ]$): $p_o(1) = 0.2$, $p_o(2) = 0.25$, $p_o(3) = p_o(4) = 0.4$, $p_o(5) = 0.6$. We also have the following strata defined in terms of number of units defeated: { 0, 1 }, { 2, 3, 4 }, and { 5 }. We order our probabilities as follows: $q_o(1) = 0.8 \geq q_o(2) = 0.75 \geq p_o(5) = q_o(3) = q_o(4) = 0.6 \geq p_o(3) = p_o(4) = q_o(5) = 0.4 \geq p_o(2) = 0.25 \geq p_o(1) = 0.2$.

The first stratum must have zero or one unit defeated. Thus our mode for the first stratum is $q_o(1) \cdot q_o(2) \cdot p_o(5) \cdot q_o(3) \cdot q_o(4)$ (i.e., outcomes $O_1=0$, $O_2=0$, $O_5=1$, $O_3=0$, $O_4=0$), with a probability equal to $(0.8)(0.75)(0.6)^3 = 0.1296$. The second stratum must have two, three, or four units defeated and the mode is $q_o(1) \cdot q_o(2) \cdot p_o(5) \cdot q_o(3) \cdot p_o(4)$, with a probability equal to $(0.8)(0.75)(0.6)^2(0.4) = 0.0864$. In this case, we "skipped" outcome $O_4=0$ with probability 0.6 and selected outcome $O_4=1$ with probability 0.4 so that we would have *at least* 2 units defeated for this strata. Note that an equally likely selection would be $q_o(1) \cdot q_o(2) \cdot p_o(5) \cdot q_o(4) \cdot p_o(3)$. The third stratum must have five units defeated and the mode is $p_o(5) \cdot p_o(3) \cdot p_o(4) \cdot p_o(2) \cdot p_o(1)$, with a probability equal to $(0.6)(0.4)^2(0.25)(0.2) = 0.0048$.

### *Interpreting the Results of Conventional Runs Using Stratified Inputs*

If we wish to obtain an output measure from the theater-level conventional model that we wish to average across all possible outcomes (which is the sort of thing we normally do in our simulation models), we need to construct a weighted average from the *ns* runs conducted using the theater model. The weight assigned to the output measure from each run $k$ would be the total likelihood of all events within stratum $k$, $k = 1, \ldots, ns$. If it is possible to enumerate all of the possible outcomes (*nt* sufficiently small), this likelihood can be computed directly. If *nt* is too large, we can conduct a simple Monte Carlo estimation of the probability $p_k$ that an event chosen at random falls within stratum $k$, $k = 1, \ldots, ns$. This is the straightforward process of estimating the vector $\{ p_1, \ldots, p_{ns} \}$ from a multinomial distribution.

We can return to the previous example to illustrate an exact computation of the likelihood of all events within a stratum. Recall that the strata were defined in terms of number of units defeated: { 0, 1 }, { 2, 3, 4 }, and { 5 }. The probability that 0 units are defeated is $P\{0\} = q_o(1) \cdot q_o(2) \cdot q_o(3) \cdot q_o(4) \cdot q_o(5) = 0.0864$. There are $\binom{5}{1} = 5$ possible outcomes leading to 1 unit destroyed; they are:

$p_o(1) \cdot q_o(2) \cdot q_o(3) \cdot q_o(4) \cdot q_o(5)$, $q_o(1) \cdot p_o(2) \cdot q_o(3) \cdot q_o(4) \cdot q_o(5)$, $q_o(1) \cdot q_o(2) \cdot p_o(3) \cdot q_o(4) \cdot q_o(5)$, $q_o(1) \cdot q_o(2) \cdot q_o(3) \cdot p_o(4) \cdot q_o(5)$, $q_o(1) \cdot q_o(2) \cdot q_o(3) \cdot q_o(4) \cdot p_o(5)$

with a total probability of $0.0216+0.0288+0.0576+0.0576+0.1296 = 0.2952$. Thus the total likelihood of the events in the first stratum is $0.0864 + 0.2952 = 0.3816$.

The calculations for $P\{2\}$, $P\{3\}$, and $P\{4\}$ are messy (more combinations) but straightforward. The likelihoods are $P\{2\} = 0.3612$, $P\{3\} = 0.2012$, and $P\{4\} = 0.0512$, for a total likelihood of 0.6136. The likelihood of the third stratum is $P\{5\} = 0.0048$.

*Adjustments*

In practice, several cases may arise where it is desirable to make some adjustments to the basic model. We describe some of them here.

***a. Likelihood of any realization within a strata being too small.*** In some cases, the total likelihood of any realization from a particular strata may be too small to justify further consideration. An example of this is the third strata ( {5} ) discussed in the previous paragraph. A probability of less than 0.01 is likely small enough to ignore in our theater level modeling (this threshold is, of course, a matter of judgment) In cases such as this, we may wish to simply run the conventional theater model with the modes from the more likely (in the example, the first and second) strata.

***b. The modes from two strata are outcomes that are adjacent to one another.*** It is possible that the modes from two strata are at the boundary of their respective strata, next to the same partition, and thus adjacent to one another in terms of an ordered outcome space. An example of this is also provided in the previous paragraph, where the modes from the first two strata are adjacent to one another in terms of units defeated (one unit defeated in the first stratum and two in the second). In order to reinforce our second assumption (different results from different strata), we may wish to make a different selection from one stratum or the other in order to avoid similar results. Two possible adjustments come to mind.

(1) The first adjustment is to select the next highest likelihood from within either stratum that does not provide the same number of units defeated as does the mode. In our example, we would choose either an outcome of zero units defeated from the first stratum or three or four units defeated from the second stratum. The most likely outcome where zero units are defeated is $q_o(1)\cdot q_o(2)\cdot q_o(3)\cdot q_o(4)\cdot q_o(5) = 0.0864$. The most likely outcome where three or four units are defeated is $q_o(1)\cdot q_o(2)\cdot p_o(5)\cdot p_o(3)\cdot p_o(4) = 0.0576$. Since $0.0864 > 0.0576$, we could choose the outcome of zero units defeated from the first stratum and keep the outcome we previously computed (two units defeated) for the second stratum.

(2) The second possible adjustment is to define partitions such that there are "gaps" between the strata. In our previous example, we might define significantly different outcomes coming from zero or one units defeated, three or four defeated, and five defeated, where the outcome of two units defeated may be an ambiguous case leading to either the same result as { 0, 1 } or { 3, 4 } defeated units. This approach may be more realistic, as the "transitional cases" at the boundaries of the exhaustive strata may lead to theater outcomes that are not as clear cut as those nearer the center of any particular stratum. The only drawback to this approach is the fact that the total likelihood of drawing results from any of the strata will not equal one.

*Repeated Exchanges*

Until now, we have assumed that there is essentially only one nuclear exchange of interest. In other words, we have assumed that the nuclear weapons will be employed during a relatively small timeframe within the overall theater battle, and that the theater battle will be conventional thereafter (at least for the duration of the conflict to be simulated). However, it is possible that a scenario may call for repeated exchanges of nuclear weapons. We can handle each exchange by defining the outcomes through binary variables and stratifying the outcome space as explained above. However, constructing an experimental plan with a reasonable number of runs of the theater model becomes difficult. The difficulty rises from the total number of possible combinations of individual exchange outcomes, even if only a few strata are chosen for each exchange. For example, only three exchanges with only three significantly different outcomes (strata) predicted per exchange will lead to $3^3 = 27$ different possible outcomes after all three exchanges. It is probably too expensive to execute this many runs of a theater-level simulation model.

To handle such a situation, we begin by determining the probability of defeating each theater-level unit and partitioning the set of all possible outcomes as explained previously. We can diagram

the 27 possible outcomes for our example as shown below in Figure 1. If 27 runs are too many to execute on our theater level simulation, then we must select a smaller subset of the 27 outcomes to actually use. The question is, of course, which subset do we pick? A stochastic simulation will randomly select paths through the "tree" (Figure 1) by selecting individual exchange outcomes randomly according to their likelihoods. When a stochastic simulation is run multiple times, the paths with a high probability of occurrence will be selected multiple times and the paths with a low probability of occurrence will be selected infrequently if at all. The result is a weighted set of outcomes that can be used to estimate the distribution of the actual outcome after three exchanges. In our case, we cannot even afford to run the model once for each possible outcome, much less multiple times. However, we have the same objective of trying to determine a set of outcomes corresponding to particular paths that can be weighted to estimate the distribution of the actual outcome after three exchanges.

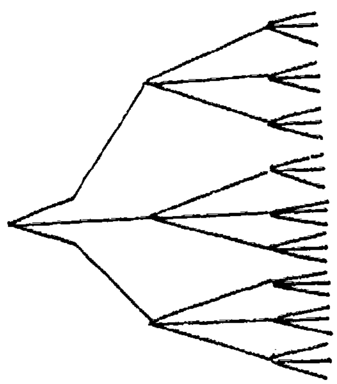

Figure 1. Possible Outcomes from Three Exchanges with Three Strata Each

Following the example diagrammed in Figure 1, let us label the strata at each exchange as high (H), medium (M), and low (L) corresponding to some exchange outcome along some measure (e.g., total units defeated). We can bound the outcome using the extreme choices at each decision point in our tree; i.e., HHH for an upper bound and LLL for a lower bound. We can also choose an intermediate outcome (MMM) in this case by choosing the intermediate result at each decision point (note that there may not always be a clearly defined "middle"). Beyond this, we need some sort of rationale for selecting particular outcomes out of the 27 possible. It is important to note that the variables are nested. For example, the middle outcome from a second strike following a high outcome from the first exchange (HM) will be different from the middle outcome from a second strike following a low outcome from the first exchange (LM), because the force strengths surviving the first exchange (and thus the subsequent theater battle before the second exchange) are significantly different.

Several approaches come to mind, both qualitative and quantitative. Qualitative approaches will choose outcomes according to the strata; for example, alternating sequences such as HML, LMH, and MLH could be chosen.

Quantitative approaches will look at the probability assigned to each stratum. For purposes of illustration, assume that the probability for the outcomes ( H, M, L ) are ( .2, .5 .3 ) respectively, and that the probability for H, M, and L are identical for each of the three exchanges (in reality, this would be unlikely but it suffices for illustration). We select our runs according to their probabilities. For example, the most likely outcome will be MMM with probability $(.5)^3 = 0.125$. The next most likely are LMM, MLM, and MML with probability $(.5)^2(.3) = 0.075$, etc. We can concentrate on choosing the outcomes with the greatest likelihood (possibly in addition to the bounds HHH and LLL).

Interpreting the output becomes more difficult when we run only a subset of all possible outcome strata. In our standard experimental plan, we run all possible outcome strata and weight the result with the probability associated with the strata. If we do not make any adjustments (such as defining non-adjacent strata), the probabilities of a realization coming from a stratum will sum to 1. When we select a subset of outcome strata, the associated probabilities will not sum to 1. We recommend normalizing the probabilities associated with the outcomes selected and proceeding accordingly. An example should make this clear.

*Repeated Exchanges -- an Example*

Suppose we have three exchanges with three significantly different outcomes (strata) H, M, L with probabilities .2, .5, .3 respectively as stated previously. A possible selection scheme might be the following.

(1) Select the upper and lower bounds HHH and LLL. The associated probabilities are HHH = $(.2)^3 = 0.008$ and LLL = $(.3)^3 = 0.027$.

(2) Select the middle (qualitative) or modal (quantitative) outcome. In this case, they are the same (MMM) with probability $(.5)^3 = 0.125$.

(3) Select the next most likely outcomes LMM, MLM, and MML. The associated probabilities are equal at $(.5)^2(.3) = 0.075$. Alternatively, some type of alternating strata sequence could be used.

This forms a subset of 6 outcomes out of the 27 possible. The total probability of a realization coming from any of the 6 selected outcomes is 0.008 + 0.027 + 0.125 + (3)(0.075) = 0.385. The normalized probabilities are therefore:

$$HHH = \frac{0.008}{0.385} = 0.021$$

$$LLL = \frac{0.027}{0.385} = 0.070$$

$$MMM = \frac{0.125}{0.385} = 0.325$$

$$LMM, MLM, MML = \frac{0.075}{0.385} = 0.195.$$

This sums to 1.001 due to rounding error.

In this example we would execute six runs of the theater-level simulation model, selecting realizations from the strata associated with each exchange as indicated above (for example, MLM would select from the middle stratum for the first and third exchange, and the lower stratum in the second). The theater-level model output associated with each realization selected can be weighted with the normalized probability of occurrence.

Note that we only account for 38.5 percent of the possible outcomes in terms of probability. As a result, our estimates made from only six runs will not be as good as those produced from a larger subset from the 27 possible.

*Averaging the Results*

To continue our example, suppose that an outcome for some particular measure from a theater conventional model such as FORCEM was 125 for a run using input from the first stratum, 75 for a run from the second stratum, and 25 for a run from the third stratum. An average value for this measure would be derived from weighting the output from a given run with the total probability of any realization coming from within the stratum. In our example, we have (125)(.3816) + (75)(.6136) + (25)(.0048) = 93.84. This value, along with the range of values produced by the three different runs (summarized perhaps with a weighted variance or other statistic), should be much more meaningful than the value obtained by running FORCEM only for some arbitrarily chosen input set for the nuclear exchange outcome.

However, a word of caution is necessary. We started with the assumption that there is more than one significantly different outcome in the theater context; in our example, there were three. A single summary measure, such as the average, does not reflect this reality. Even a sample average

and variance will not inform a decisionmaker about the possible outcomes along with their associated probabilities. Since the total number of runs of the theater conventional model will be (by necessity) small, we recommend reporting *all* of the results, accompanied perhaps with a summary measure. In cases of tactical nuclear warfare, we are often concerned with relatively unlikely events (such as the exchange itself) that nevertheless have a very significant impact. Averaging obscures this fact and can lead a decisionmaker astray.

*Summary*

Using a deterministic, expected value approach to model a real-world situation such as theater-level combat poses problems in selecting input data. A deterministic simulation demands a single input data set for a model run, while the data may have to represent a process that is inherently stochastic. An example is provided in this paper. The results of a tactical nuclear exchange within a theater is inherently stochastic, driven by random events such as target acquisitions. An "average" exchange outcome cannot properly be defined; an average fails to exist in subset selection problems (for example, if 20 units out of 50 are acquired on the average, *which* 20 are to be selected as acquired in the deterministic model?) Even where averages can be defined, they fail to reflect important variations in possible outcomes that may make a difference between winning and losing the war in a theater simulation.

Ideally, a theater-level stochastic model would be used to properly reflect uncertainties inherent in the data and processes represented by the model. However, the current state of the art in hardware and software only permit us (at present) to model combat at the theater in a deterministic, low-resolution mode. Thus, we must reconcile the need to provide an input to these deterministic models with the reality of random outcomes.

If there are approximately $10^4$ potential nuclear targets in a theater, there are $2^{10^4}$ possible outcomes that can occur in terms of the defeat or failure to defeat each potential target. Even if we look only at the defeat or failure to defeat the low resolution aggregate units represented in our theater model, we still have on the order of $2^{10^2}$ possible outcomes. A classical experimental design approach that requires at least one run per variable obviously cannot be applied. The challenge, then, is to construct a plan that minimizes the number of different input data sets yet fully reflects the range of possible outcomes of the theater nuclear exchange.

This paper outlines an approach to constructing such an experimental plan. We begin with the probability of defeating a potential nuclear target $p_{defeat}(i)$ and determine from that the probability of defeating the aggregate units represented in our theater model (such as divisions). We can characterize all possible outcomes of the exchange as sets of binary variables, where each binary variable reflects the defeat or failure to defeat each unit. We then partition the outcome space into strata such that outcomes from different strata lead to significantly different results in the theater battle, and all significantly different outcomes are included in some stratum. Our experimental plan consists of a nuclear exchange realization from each strata that corresponds to the most likely outcome within that stratum. The theater-level model is run using the experimental plan to determine the appropriate input data set to use to reflect the outcome of a theater nuclear exchange.

*Directions for Future Research*

The techniques outlined in this paper form only a start at trying to resolve the issue of how to handle uncertainty in input to large, complex expected value models. They are presently limited to input processes that can be summarized in a reasonable number of binary variables, where it is possible to make a judgement about the type of expected value model output given sets of similar input realizations. Nevertheless, it is a step in the right direction. At present, it is not infrequent to find studies based on a single model run per input scenario, without any estimate of the variability possible in the results obtained.

Possible future research topics include extending the techniques to proceeses that can be expressed in various states, the number of such states exceeding two. Better ways of estimating partitions of the sample space may also be developed. A very realistic case in many theater scenarios involves repeated realizations of random processes (in the context of the nuclear exchanges discussed in the paper, this would imply many small weapon exchanges over a relatively long period of time). At present, we have no satisfactory way of handling this situation. Robust experimental plans that can provide meaningful results over a large number of repeated realizations will be be necessary to model such scenarios.

# APPENDIX A
# REFERENCES


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# APPENDIX B
## NOTATION

$B_i$ — A binary variable denoting the defeat / failure to defeat outcome of a nuclear exchange against a targetable unit, where $B_i = 1$ if the unit is defeated; 0 otherwise.

$D_n$ — A binary variable denoting the defeat / failure to defeat outcome of a nuclear exchange against a division for the $n$th Monte Carlo replication, where $D_n = 1$ if the set of numbers $B_i$ drawn correspond to a division being defeated, 0 otherwise.

H,M,L — A qualitative measure of the nuclear exchange outcome along some measure (e.g., total units defeated). H stands for high, M for medium, and L for low. When ordered, e.g., HLM, the first letter represents the outcome of the first exchange, the second the outcome of the second exchange, etc.

$K_s$ — A variable which denotes the $s$th possible min cut sets (in this case, a min cut set is any set of $m$-$k$+1 units).

*ndiv* — The number of divisions.

*ns* — The number of strata selected for a particular nuclear exchange.

*nt* — The number of targetable subunits in the theater.

$O_j$ — A binary variable used to define the outcome of the nuclear exchange with respect to division $j$, $j = 1, \ldots, ndiv$. $O_j = 1$ with probability $p_{defeat}(div\ j)$ if division $j$ is defeated; 0 otherwise.

$\phi(\underline{O})$ — A binary function of the random variables $\underline{O}$, defined such that $\phi(\underline{O}) = 1$ if the commander's objective (for the theater-wide nuclear exchange) is met; 0 otherwise.

$P_r$ — A variable which denotes the $r$th possible min path sets (in this case, a min path set is any set of $k$ units).

$p_{defeat}(div\ j)$ — Probability that division $j$ defeated ] for $j = 1, \ldots, ndiv$.

| | |
|---|---|
| $p_o(i)$ | The probability that $P[\ O_i = 1\ ]$; that is, the probability that division $i$ is defeated. |
| $(\ x_j\ ) \amalg (\ x_i\ )$ | For 2 units, $(\ x_j\ ) \amalg (\ x_i\ ) = 1 - (\ 1 - x_i\ )(\ 1 - x_j\ )$. |
| $\coprod_i X_i$ | For $n$ units, $\coprod_i X_i = 1 - \prod_i (1 - X_i)$ |
| $\binom{n}{k}$ | The binomial coefficient. $\binom{n}{k}$ is defined as $\frac{n!}{k!(n-k)!}$. |

## APPENDIX C

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